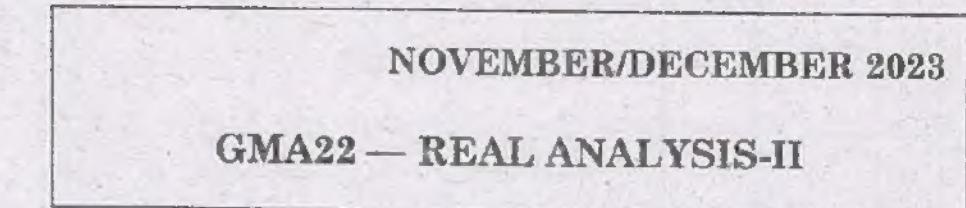


19. Prove that  $u$  and  $v$  be two real-valued functions defined on a subset  $S$  of the complex plane. Assume also that  $u$  and  $v$  are differentiable at a interior point  $c$  of  $S$  and that the partial derivatives satisfy the Cauchy-Riemann equations at  $c$ . Then the function  $f = u + iv$  has a derivative at  $c$ . Moreover  $f'(c) = D_1 u(c) + iD_2 v(c)$ .
20. A quadric surface with center at the origin has the equation  $Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1$ . Find the lengths of its semi-axes.
- 



Time : Three hours

Maximum : 75 marks



SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Lebesgue Integral.
2. Define Monotonic Sequences.
3. Define Riemann-integrable.
4. Define Measurable Functions.
5. Define Orthogonal Systems.
6. Define Dirichlet Integrals.
7. Directional Derivative.
8. Write down Taylor's formula.
9. Prove that  $u + iv$  is a complex-valued function with a derivative at a point  $z$  in  $\mathbb{C}$  if  $|f'(z)| = |f'(z)|^2$ .
10. State Inverse Function theorem.

**SECTION B — (5 × 5 = 25 marks)**

Answer ALL questions.

11. (a) Prove that  $f \in U(I)$  and  $g \in U(I)$ , and if  $f(x) = g(x)$  almost everywhere on  $I$ , Then  $\int f = \int g$ .

Or

- (b) Prove that  $f \in U(I)$  and  $\{s_n\}$  and  $\{t_m\}$  be two sequences generating  $f$ . Then  $\lim_{n \rightarrow \infty} \int s_n = \lim_{m \rightarrow \infty} \int t_m$ .
12. (a) Prove that  $f \in M(I)$  and if  $|f(x)| \leq g(x)$  almost everywhere on  $I$  for some nonnegative  $g$  in  $L(I)$ , then  $f \in L(I)$ .

Or

- (b) Prove that  $f \in L(I)$  and if  $f$  is bounded almost everywhere on  $I$ , then  $f^2 \in L(I)$ .

13. (a) Prove that  $g$  is of bounded variation on  $[0, \delta]$ ,  
Then  $\lim_{\alpha \rightarrow +\infty} \frac{2}{\pi} \int_0^\delta g(t) \frac{\sin \alpha t}{t} dt$  is equal to  $g(0+)$ .

Or

- (b) Show that  $x = \pi - 2 \sum_{n=1}^{\infty} \frac{\sin nx}{n}$  if  $0 < x < 2\pi$ .

14. (a) Prove that  $S$  be an open connected subset of  $R^n$ , and let  $f : S \rightarrow R^m$  be differentiable at each point at each point of  $S$ . If  $f'(c) = 0$  for each  $c$  in  $S$ , Then  $f$  is constant on  $S$ .

Or

- (b) State and Prove that Taylor's theorem.

15. (a) Prove that  $A$  be an open subset of  $R^n$  and  $f : A \rightarrow R^n$  is continuous and has finite partial derivative  $D_i f_i$  on  $A$ . if  $f$  is one-to-one on  $A$  and if  $J_f(X) \neq 0$  for each  $x$  in  $A$ , then  $f(A)$  is open.

Or

- (b) State and prove Implicit function theorem.

**SECTION C — (3 × 10 = 30 marks)**

Answer any THREE questions.

16. State and Prove Lebesgue dominated convergence theorem.
17. State and Prove Riesz- Fischer theorem.
18. State and Prove Weierstrass approximation theorem.